ME/SE 740

Lecture 15

Denavit-Hartenberg Parameters

We will now go through and assign coordinate frames for each link in a kinematic chain using the Denavit-Hartenberg procedure. Refer to the figure below for details:

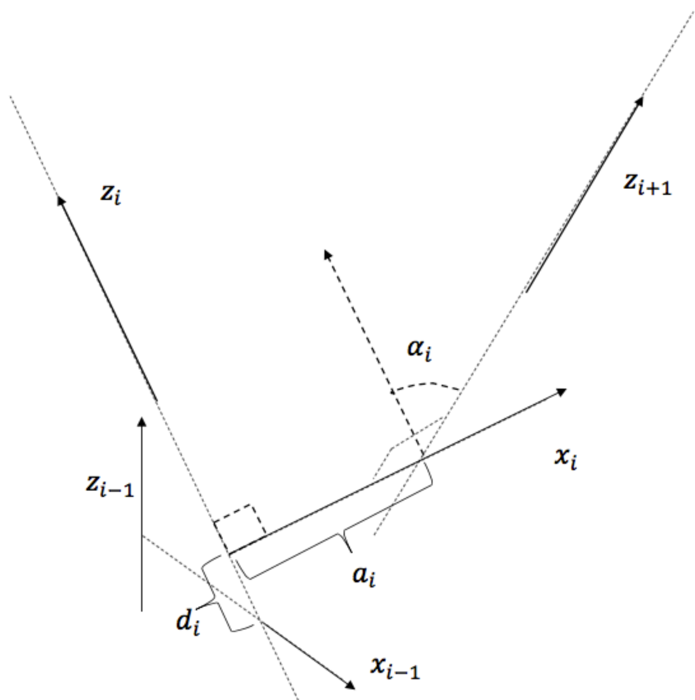

Figure 1: **Denavit-Hartenberg Parameters**

Consider joint i.

- Let d_i be the signed distance from the point where x_{i-1} intersects z_i, to the point where x_i intersects z_i.
- θ_i is an angle (about z_i) measured in the counterclockwise sense between the x_{i-1} axis and the x_i axis.
- a_i is the link length (distance between link i and $i+1$)
- α_i is the twist about x_i of z_i into z_{i+1}

The four parameters $a_i, \alpha_i, d_i, \theta_i$ are called the Denavit-Hartenberg parameters. In the example below showing two consecutive links, (links rotate about bearing A and bearing B axes) for link A, $a = 7''$, and $\alpha = 45°$.

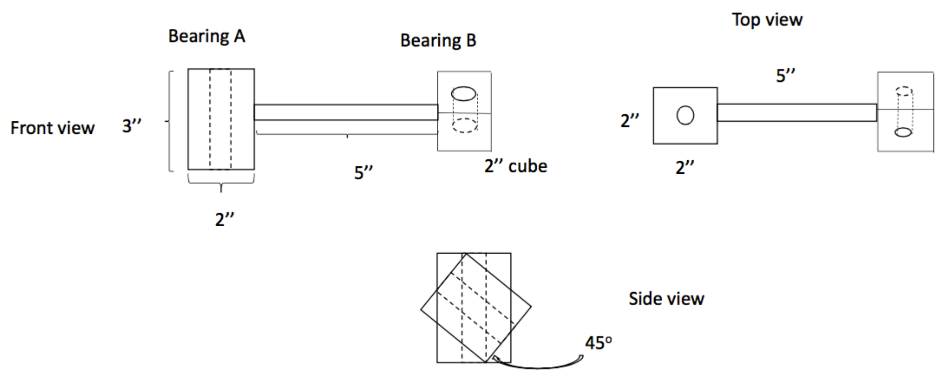

Figure 2: **Two Link Manipulator**

When there are arbitrary choices to be made make the parameter assignments 0 unless there are reasons to do otherwise.

E.g., when succeeding z-axes are parallel choose x_{i-1} to make $d_{i-1} = 0$ (for the case of revolute joints).

First and Last links

Given a serial mechanism with axes 1 through n, a_1 through a_{n-1} and α_1 through α_{n-1} are denined as above. Choose $\alpha_0 = 0, a_0 = 0$ (this means that z_1 coincides with the z-axis of the base frame). Similarly, $\alpha_n = 0, a_n = 0$ (or don't bother to define them).

The link offset d_i and joint angle θ_i are well defined for joints 2 through $n-1$. If joint 1 is revolute, let θ_1 have arbitrary zero position and $d_1 = 0$. If joint 1 is prismatic we let d_1 have arbitrary 0 position and $\theta_1 = 0$. Similar for joint n.

Summary

If successive z-axes are parallel, the origin is chosen to make link offset 0 for the next link whose coordinate system is to be defined:

z_k axis is axis of rotation of the k^{th} joint.
x_k axis is aligned with normal direction from z_k to z_{k+1}
for intersecting z-axes, z_k intersects z_{k+1} and $x_k = z_k \times z_{k+1}, \quad y_k = z_k \times x_k$.

Example 1

Consider the 6-link manipulator (referred to as the "elbow manipulator) shown below. It is an abstraction of the "Cincinnati Milacron T3". We would like to obtain the Denavit-Hartenberg parameters:

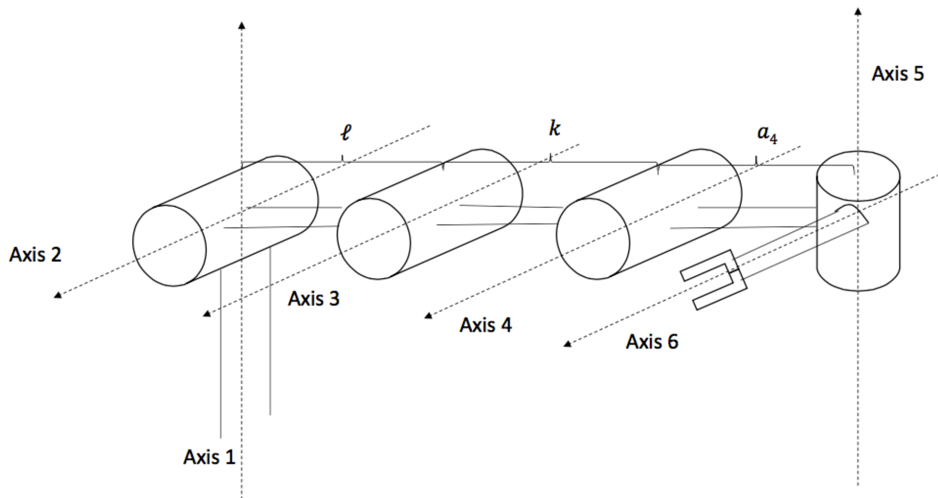

Figure 3: **Elbow Manipulator**

Some observations about the geometry and axis properties:

- Axis 1 is perpendicular (⊥) to axis 2 and they intersect
- Axis 2 is parallel (||) to axis 3 and || to axis 4
- Axis 5 and axis 6 are ⊥ to each other and they intersect

D. H Parameters

i	a_i	α_i	d_i
1	0	90°	0
2	ℓ	0	0
3	k	0	0
4	a_4	−90°	0
5	0	90°	0
6	0	0	0

Example 2

Consider now the 6-link manipulator depicted below, it is an abstraction of the PUMA 560 robotic manipulator (Unimation ⟶Westinghouse).

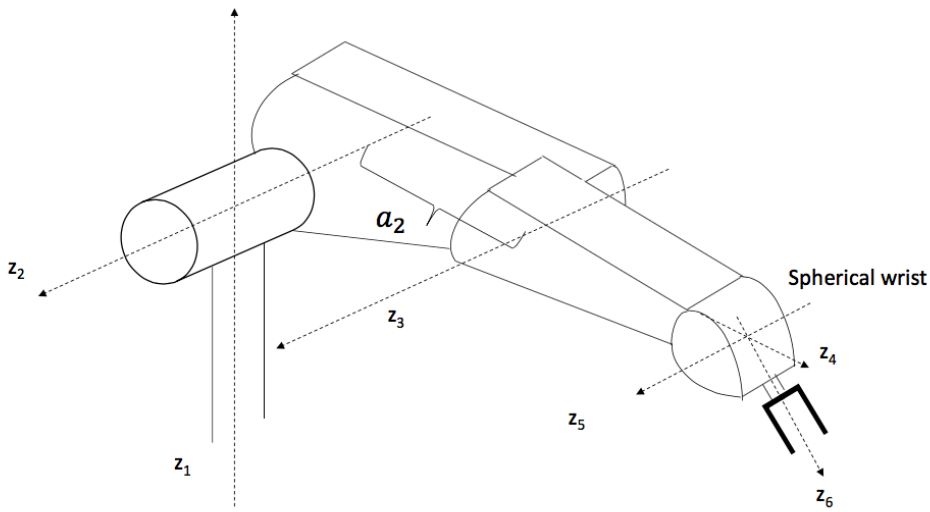

Figure 4: **Elbow Manipulator**

Some observations about the geometry and axis properties followed by a figure that depicts the PUMA 560 "Forearm":

- Axis 1 is $\perp$ to axis 2 and they intersect
- Axis 2 is $||$ to axis 3
- Axis 3 $\perp$ to axis 4 but they do not intersect
- Axes 4, 5, and 6 intersect at a point (end effector is called a "spherical wrist") and axis 4 is $\perp$ to axis 5, and axis 5 is $\perp$ to axis 6

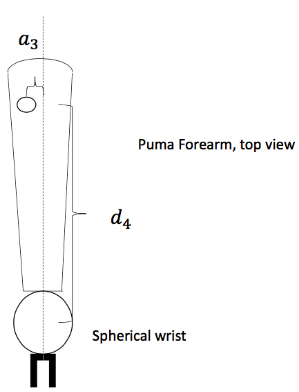

Figure 5: **PUMA 560 Forearm**

D. H Parameters

i	a_i	α_i	d_i
1	0	90°	0
2	a_2	0	0
3	a_3	90°	d_3
4	0	-90°	d_4
5	0	90°	0
6	0	0	0